1. A particle A of mass 4m is moving with speed 3u in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass 3m moving with speed 2u in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is 4eu.

(a) Show that
$$e = \frac{3}{4}$$
.

(b) Find the total kinetic energy lost in the collision.

(4) (Total 9 marks)

(5)

(5)

(5)

- 2. A particle *P* of mass 2m is moving with speed 2u in a straight line on a smooth horizontal plane. A particle *Q* of mass 3m is moving with speed *u* in the same direction as *P*. The particles collide directly. The coefficient of restitution between *P* and *Q* is $\frac{1}{2}$.
 - (a) Show that the speed of Q immediately after the collision is $\frac{8}{3}u$.
 - (b) Find the total kinetic energy lost in the collision.

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(c) Calculate the range of values of e for which there will be a second collision between P and Q.

(7) (Total 17 marks)

M2 Collisions

1. (a)

$$3u \rightarrow 2u \rightarrow 3m$$

$$4m \rightarrow 3m \rightarrow 2u \rightarrow 3m$$

$$x \rightarrow 2u \rightarrow 3m \rightarrow 3m$$

$$x \rightarrow 2u \rightarrow 3m \rightarrow 3m$$

$$x \rightarrow 2u \rightarrow 3m \rightarrow 3m$$

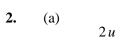
$$y = 4eu \rightarrow 2m \rightarrow 3m \rightarrow 3m$$

$$12mu + 6mu = 4mx + 12meu \qquad B1$$

$$NEL \qquad 4eu - x = eu \qquad M1A1$$
Eliminating x to obtain equation in e
$$DM1$$
Leading to $e = \frac{3}{4}(*) \qquad cso \qquad A1 \qquad 5$

(b)
$$x = 3eu \text{ or } \frac{9}{4}u \text{ or } 4.5u - 3eu$$
 seen or implied in (b) B1
Loss in KE $= \frac{1}{2}4m(3u)^2 + \frac{1}{2}3m(2u)^2 - \frac{1}{2}4m\left(\frac{9}{4}u\right)^2 = \frac{1}{2}3m(3u)^2$ M1A1ft
ft their x
 $= 24mu^2 - 23\frac{5}{8}mu^2\frac{3}{8}mu^2 = 0.375mu^2$ A1 4

[9]



$$2u$$

$$2m$$

$$2m$$

$$3m$$

$$3m$$

$$3m$$

$$4mu + 3mu = 2mx + 3my$$

$$NEL \quad y - x = \frac{1}{2}u$$

$$Solving to \ y = \frac{8}{5}u^{*}$$

$$cso \quad M1 \text{ A1}$$

(b)
$$x = \frac{11}{10}u$$
 or equivalent B1
Energy loss $\frac{1}{2} \times 2m \left((2u)^2 - \left(\frac{11}{10}u\right)^2 \right) + \frac{1}{2} \times 3m \left(u^2 - \left(\frac{8}{5}u\right)^2 \right)$ M1 A(2,1,0)
 $= \frac{9}{20}mu^2$ A1 5

(c)	$\frac{8}{5}u$				
	$LM \frac{24}{5}mu = 3ms + mt$	M1 A1			
	NEL $t-s = \frac{8}{5}eu$	B1			
	Solving to $s = \frac{2}{3}u(3-e)$	M1 A1			
	For a further collision $\frac{11}{10}u > \frac{2}{5}u(3-e)$	M1			
	$e > \frac{1}{4}$	ignore $e \leq 1$	A1	7	
	•			[1	17]

- 1. (a) Very few candidates had a problem with the two equations required. The most common errors were still inconsistency in signs and getting the ratio the wrong way round for the impact law. This is a very costly error leading to the loss of most marks for this part of the question.
 - (b) Some candidates struggled with the algebra here. The correct values for the speeds of *A* and *B* immediately after the collision were usually used, but, especially when candidates found the change in kinetic energy of the particles separately and then combined their answers, they failed to subtract energies correctly. A significant number managed to make mass and velocity disappear from their final answer having successful worked through the algebra to find the kinetic energy lost.
- 2. This question was generally well understood and answered. Most errors were caused by poor presentation leading to carelessness. Candidates who kept all the velocities in the direction of the original velocities usually fared better than those who reversed one or more velocity. The clearest solutions included clearly annotated diagrams which made the relative directions of motion very clear. In the weaker solutions it was sometimes difficult to work out the candidate's thoughts about what happened in each collision -the question did not give them names for the speeds after the initial collision and this gave rise to problems for some candidates who often gave the same name to more than one variable. Candidates with an incorrect or inconsistent application of Newton's Experimental Law lost a lot of time trying to obtain the given answer for the speed of Q after the first collision. In part (b) although most candidates attempted to form a valid expression for the change in kinetic energy, the m and u^2 were too often discarded along the way.

In part (c), and to a lesser extent in part (a), the tendency to want to solve simultaneous equations by substitution, rather than by elimination, produced untidy and unwieldy expressions which often led to arithmetical errors; a shame when the original equations were correct. Most candidates interpreted the final part correctly, although too many wanted to substitute 11/10u rather than tackle an inequality -it was clear that many candidates were not confident in setting up an inequality. Some problems did occur where students assumed the reversal of the direction of motion of Q following the collision but failed to take account of this in setting up their inequality.